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# "Marketing Strategies for Products with Cross-Market Network Externalities"

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# MARKETING STRATEGIES FOR PRODUCTS WITH CROSS-MARKET NETWORK EXTERNALITIES[1]

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Abstract

This paper discusses marketing strategies for a manufacturer of a composite product (i.e., a product sold in two parts to two separate types of consumers, which has greater value when used jointly by both consumers). It is assumed that expected sales of one product increase a different type of consumer's willingness to pay for the other product. This is described as a "cross-market network externality". Using a parsimonious model, I characterize solutions for a monopolist and for variations on Bertrand-Nash competition with differentiated products. The results demonstrate that in the presence of the cross-market network externality, it is optimal for the price of one product to increase relative to the price of the other product (by comparison with expected prices in the absence of the externality effect). In competition between firms, it is individually rational for each firm to attempt to maximize its own cross-market network externality. However (and counter-intuitively), as all firms strive to increase the cross-market externality related to their products, a worse industry-wide solution results. In other words, the existence of firm-specific cross-market network externalities in Bertrand-Nash competition creates a form of "Prisoner's Dilemma". In some multi-firm competitive markets, compatibility between products is a design choice (for example, compatibility between word processors such as Microsoft Word and WordPerfect). Normally, increasing compatibility between products increases the substitutability of the products, often resulting in more competition and lower profits (absent other effects). In a market with cross-market externalities, however, increased compatibility leads to increased profits. The model utilized in this paper has applications to industries as diverse as television, Internet web portals and certain types of Internet browser software. It is expected that the predictions of the model will be tested on a data set from the software industry.


Keywords: Internet, Software, and Network Externality

## 1 Introduction

Consider the introduction of advertiser-supported network television ("TV") in the United States in the past, or High Definition TV ("HDTV") in the present. At the outset, consumers will be reluctant to purchase televisions unless they are assured that content will be available (e.g., that TV stations will broadcast news and entertainment programs). Similarly, advertisers will be unwilling to pay for advertisements, unless they can be assured that consumers will purchase (and use) the TV sets. It is reasonable to suppose (as a crude first approximation) that the amount advertisers will pay has some correlation to the number of TV sets sold. It is also reasonable to assume that the number of TV sets sold may partially depend on the amount of programming that will be broadcast. However, advertisers and consumers are very different types of customers.

The more TV programming that is broadcast, the more TV sets that will be sold. Additionally, the more TV sets sold, the more (and/or higher priced) TV advertising that will be sold. Hence the networks will spend more on programming. Television advertising and TV sets, however, are two separate goods sold to two separate consumer groups. Nonetheless, a broadcaster might subsidize the sale of TV sets to generate a larger TV audience, so that more commercials could be sold (and/or at a higher price). Similarly, a TV set manufacturer might subsidize television programming.

The development of HDTV involves just such cross-subsidies. For example, the May 10, 1999 edition of The New York Times (Brinkley (1999)) reported that:

*"In an effort to jump-start the nation's sluggish transition to digital television, the Mitsubishi Electric Corporation, a major maker of television sets, has agreed to underwrite CBS's costs to broadcast most of the network's prime-time schedule in high definition beginning next fall…Mitsubishi hopes that it will 'eliminate the chicken-and-egg problem bothering the industry'…Other networks have quietly received limited subsidies from set makers like Sony and Matsushita Electric Industrial's Panasonic to broadcast shows in HDTV…Network executives have long said they believe television*

*manufacturers have the most to gain from widespread acceptance of high-definition TV; for it is they who will make money from the sale of all those new HDTV sets…Mitsubishi will spend 'millions of dollars' on this deal…"*

The Wall Street Journal (May 10, 1999), reporting on the same Mitsubishi-CBS arrangement, noted:

*"Though such arrangements occurred during the advent of radio and color-TV broadcasts, the agreement is the first time in the nascent era of digital broadcasting that a manufacturer will pay for shows to drive demand for its equipment. Mitsubishi will spend a little over $10 million to translate CBS's prime-time shows from film into digital code."*

This type of phenomenon is not limited to the television industry. Consider the market for web site management software, such as Microsoft FrontPage. The software is sold in two parts. The first part (the Web Composer software) is sold to people who design and build web sites. The second part of the software is sold to the Internet Service Providers (ISP's) who host web sites. Each software company designs its product to encourage the ISP and web site manager to buy these two different pieces of software from the same firm. The more Microsoft FrontPage software is sold to the web site managers, the more Microsoft will be able to sell Microsoft FrontPage Server to the ISP's.

A Plug-in (e.g., Acrobat, Shockwave, or Cult3D---Netscape listed 176 software Plug-ins as of November 1998) extends the capabilities of Internet Browsers in various ways. The general format of a Plug-in allows a Browser user to download and read a specialized type of file from a web site (for example, files in Adobe PDF). All the Internet Browser Plug-ins examined were provided free of charge, and indeed downloading free Plug-ins is now a regular daily event.

The software manufacturer gives the Plug-in to Browser users, and then sells to web site managers the software needed to generate the files. As a specific example, consider RealPlayer. RealPlayer is a software program used by consumers to see/listen to video/audio (e.g., movie clips, music, and the news) on the worldwide web. The material is broadcast as a stream of information (i.e., a computer file), and consumers need a computer program to open this file and see/hear the video/sound. The consumers receive a free software program (i.e., the Plug-in) which allows them to receive and use the RealPlayer file. The Plug-in software is free, but the computer program generating the data file is sold (for as much as $10,000 a copy) to stations and firms that want to broadcast over the web.

The more the first group uses the free Plug-in, the more the second type of software can be sold to the other group. Firms have adopted this strategy so rapidly that it is a marvel to behold---until we realize that, given the underlying mathematics of the situation (particularly the low marginal costs involved in distributing software over the Internet), this is the Nash equilibrium outcome.

I focus on an abstract market where between 1 to 2N firms compete. Each competitor sells two types of multi-attribute products (or where the number of competitors is 2N, each competitor sells one type of product). The products are sold to two different types of consumers. The products form a composite product. A manufacturer's product has increased value to the consumer, to the extent its sister product is purchased by different consumers in a separate market. This means that the willingness to pay in one market is dependent (but not equally) on sales in the other market. The inequality is important because, to the extent prices can be increased in one market (by decreasing prices in the other market), the firm has an incentive to “distort” the relative pricing of the two products, and an advantage will accrue to coordinated production.[3] The proposed model does not

[3] These results have analogies to markets such as camera/film or razors/razor blades, where firms may under-price one product to increase demand for the other product. However, the purchaser of a razor is also the purchaser of the blades. He does not have an externality in the actions of a different and unrelated user. In the market for Plug-in software, the firm purchasing the RealPlayer

perfectly reflect any specific market, but it is hoped that the intuition derived will have implications for many analogous marketing situations.

I compare two market structures: a base case of monopoly, and a Bertrand-Nash solution with differentiated products. The results are consistent and striking that firms will often under-price one product to generate sales in the other product. An examination of these results suggests that one product can be priced at almost zero, and all profit will be made on the other product, provided certain conditions are met.

The intuition for these results is that if one market has an impact on the other (via the network feedback), competitors have an incentive to charge different prices for (as an example) the server and the Plug-in---even if the marginal costs of production, etc. for these products are identical. If marginal costs are sufficiently low, manufacturers will literally give away one product (or even set a negative price[4] for one product) to increase the price/sales of the other product. It is hypothesized that manufacturers will not set a substantially negative price, however, because of the moral hazard risk (i.e., at a negative price, non-users of the product have incentives to pretend to be users). For example, a broadcaster (if the subsidy "runs" in that direction) would generally not extend the subsidy on the sale of TV sets to the point where a large negative price is attached to the TV set. A sufficiently negative price would assure that everyone buys a TV, but not that every purchaser uses it. Accordingly, for modeling purposes, I have sometimes imposed a constraint that all prices are greater than or equal to zero. Note that when the problem is formulated as a constrained optimization, the

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server for $10,000 is generally not the same firm that receives the Plug-in software. It is this externality in another consumer's actions which gives this problem its unique flavor.

[4] A negative price might take the form of redeemable coupons, other free products, or services.

value of the Lagrangian multiplier will provide useful insights into the value of market strategies that allow the non-negativity constraint to be violated (a point that will be explored in the companion empirical paper).

As we would expect in competition between firms, it is individually rational for each firm to attempt to maximize its own cross-market network externality. However (and counter-intuitively), as all firms strive to increase the cross-market externality related to their products, a worse industry-wide solution results. In other words, the existence of firm-specific cross-market network externalities in Bertrand-Nash competition creates a sort of "Prisoner's Dilemma." Further, we normally expect that increasing substitutability between products (by increasing compatibility) causes an increase in competition (and thereby decreases profits). Counter-intuitively, in markets characterized by a cross-market externality, increasing compatibility increases competition in a way that augments profits. Both of these results are presented analytically later in this paper, and an economic interpretation is provided.

## 2 Literature Review

Industries characterized by cross-market effects (such as Media, Advertising, Software, Network Television and Radio) have annual revenues of approximately $200billion/year in the United States alone (U.S. Bureau of the Census (1997), Tables 886, 903). Surprisingly, given the importance of these industries, very little research has been done which discusses cross-market effects.

Broadly speaking, my results have similarities to the network externalities literature. For example, Economides (1996) examined network externality effects in relation to invitations to enter (e.g., licensing). Katz and Shapiro (1985) looked at the role consumer expectations play in markets with network externalities and firms' decisions regarding compatibility with competitors. Farrell and Saloner (1986) looked at the implications of the installed user base in markets with network externalities. Gandal (1994) and Brynjolfsson and Kemerer (1996) demonstrated that network externalities are not merely a theoretical construct. They found clear econometric evidence supporting the existence of network externalities in the market for PC software.

This paper differs from prior papers on network effects by its focus on cross-market network externalities. Prior literature in the network externality area focused mainly on (1) how a consumer's purchase of a product increased demand for that product (e.g., each new member of a telephone network increased its value to other telephone network users), or (2) the externality effect between complementary products (e.g., hardware/PCs and software/spreadsheets) sold in the same market (i.e., to the same type of buyers).

Of course, the impact that the sales of one product have on the pricing of another product (even absent externalities) is not a new concept. This type of pricing problem arises frequently in the bundling and tying literature. See, for example, Whinston (1990), McAfee, McMillan et al. (1989), Nalebuff (1999) and Schmalensee (1979). This literature has been widely discussed in the context of the Microsoft ("MS") anti-trust case. In the MS case, one of the key questions is whether MS used its power in the operating systems software (i.e., Windows) market to dominate the applications software market (i.e., MS Word, Excel, etc. bundled as MS Office). Again, note that the focus of the bundling and tying literature is bundles sold to a single group of consumers.

My results reflecting the desirability of giving away one product to generate sales of another product are similar to those of Hanson (1997). Hanson showed that, when a firm uses the web to enhance the quality of products and services sold offline, it is often in that firm's interest to give away the online enhancement as a complementary product to the pre-existing “real-world” product. Hanson, however, focused on situations where the product sold offline and the online enhancement are both used by the same customer, rather than where the products are sold into separate markets.

Compatibility as a factor in product design has been discussed by, among others, Besen and Farrell (1994) and Katz and Shapiro (1994). The compatibility literature distinguishes between strategies in product design for vertical and horizontal compatibility. Vertical compatibility refers to the compatibility between components sold to the same consumer. For example, certain models of the IMac have an integrated monitor that is not compatible with other computers, whereas most Windows-based systems use interchangeable components. Horizontal compatibility refers to compatibility between systems (e.g., between the two word processing systems WordPerfect and Microsoft Word). Note that both horizontal and vertical compatibility reflect sales (of products or systems) to the same type of consumer. This paper contributes to the compatibility literature by characterizing the impact of cross-market compatibility.

Chaudhri (1998) examined the economics of the newspaper industry, primarily in a monopoly setting. Chaudhri found that, among other things, newspaper proprietors have an incentive (under certain circumstances) to sell newspapers below marginal costs. The lower price of the newspapers results in increased circulation, which allows advertising to be sold at a higher price. My results are clearly analogous to Chaudhri's. My analysis differs, however, in its specific focus on marketing strategy (for price, compatibility, and cross-market externality) in competitive situations.

In summary, the contributions of this paper are: (1) its focus on manipulating cross-market network externalities and compatibility as strategic variables, and (2) the counter-intuitive direction of the resulting effects. In particular, marketers need to consider how marketing programs can be designed to take advantage of these effects.

This paper is organized as follows. The next section introduces the notation, formally defines the concept of "cross-market network externality", then characterizes the monopoly solution. Next, the competitive solution with differentiated products is characterized, and a stylized description of the Internet Browser Plug-in industry is presented. The limited data available suggest that this model has predictive validity for this particular market. The final section presents managerial implications, discusses the companion empirical paper, and proposes further extensions.

## 3 The Cross-Market Network Externality and Summary of Notation

Assume $n\,(n = 1,\ldots,N)$ firms, and for the moment, assume each firm manufactures both types of products. The products of the firms are denoted as $a_n, b_n$. Consider two types of consumers. Type A consumers consume product $a$, and Type B consumers consume product $b$. Product pairs are assumed to be incompatible between manufacturers.[5] Aside from this incompatibility (which will be relaxed later in this paper), no other restrictions are placed on product attributes. Let the sales of product $b$ by firm $n$ be represented by $q_n^b$, and similarly for product $a$. The function $z(q_n^b)$ measures the increase in the willingness to pay for $a_n$ because of the network externality in the related product. So, let the network externality function, for firm $n$ which produces product $a$ (i.e.,

[5] For example, the United States and Europe use two different television standards. In a sense, the two systems are near perfect substitutes for each other, but an American TV set will not work in Europe and vice versa (the signals, etc. are incompatible).

Table 1 Summary of Notation

| Notation | Description |
|---|---|
| $z(\cdot), D^t(\cdot), p(\cdot)$ | Externality, demand and profit functions, respectively. |
| $*$ | Superscript indicating optimal solution, in price, quantity, etc. |
| $t \in \{a, b\}$ | Superscript indicating product type. |
| $n \in \{1...N\}$ | Subscript indicating number of the firm. |
| $p_n^t, q_n^t, c_n^t, F_n^t$ $C_n, y_n^t$ | Price, quantity, marginal costs, annual fixed costs, compatibility and location in the Hotelling game, respectively. |
| $k_n$ | Degree of externality for firm $n; k_n \geq 0$. |
| $x_n^t, b_n^t$ | Product characteristics and coefficients. |
| $R^t$ | Constants of demand for monopoly analysis. |
| $M^t$ | Market size for Bertrand-Nash analysis. |
| $s, m, p, e$ | Subscripts indicating separate firm, monopolist, pioneer, and entrant, as appropriate. |
| $m$ | Degree of customer uncertainty. |
| $a$ | Error term. |

$a_n$), be given by $z(q_n^b)$.[6] Therefore, Type A consumer has an externality in Type B consumer's actions. The notation for this paper is summarized in Table 1.

And to more formally motivate the discussion of the cross-market network externality, in the presence of expected sales of size $q_n^b$, let the willingness to pay for $a_n$ increase from $p_n^a(q_n^a;0)$ to:

$$(1) \quad p_n^a(q_n^a; q_n^b) = p_n^a(q_n^a;0) + z(q_n^b)$$

Note that the model is parsimonious. No conventional network effect within the product group is needed (and no diffusion effect is assumed regarding sales of product $a$). Assume that the increase in willingness to pay (because of the externality) is the same for each unit sold, regardless of the

[6] Actually, we would expect the externality to run in both directions. However, as an analytical matter we only need to be concerned with the "net" effect. So, without loss of generality, I normalize the cross-market externality in one direction to 0.

location on the demand curve. This implies that the network externality enters additively, and increases demand without changing the slope $\left( e.g., \frac{\partial p(q_n^a; q_n^b)}{\partial q_n^a} = \frac{\partial p(q_n^a; 0)}{\partial q_n^a}, \forall q_n^b > 0 \right)$. This functional form is used for tractability.

Consider purchasers of product $a$ to be industrial users (e.g., builders and creators of commercial web sites). Think of purchasers of product $b$ as more traditional end users or consumers (e.g., consumers who download Plug-ins in order to view material on a web site).

I place the following plausible restrictions on the externality function:

1. $z(0) = 0$, No expected sales produces no network externality. This is a normalization of the function and could have been done at any arbitrary level.

2. $z(q)$ is twice differentiable $\forall q \geq 0$.

3. $z'(q) \geq 0, \forall q \geq 0$, so higher expected sales can never produce a lower externality.

4. For ease of exposition, the externality function will be assumed to be linear. Intuitively, I expect that the first consumers will be the most "valuable", with subsequent customers being less valuable. As a consequence, the form of the externality function will (probably) be concave in the empirical work.

## 4 The Monopoly Equilibrium

I introduce the monopoly case first for several reasons. A number of the industries that are of interest (for example, software and broadcast television) are quasi-monopolistic situations (Windows, RealPlayer, and Adobe Acrobat all have market shares on the order of 90%). Further, many markets can be conceptualized as evolving from a first period pioneer (who is a monopolist) into a second period competitor. As we will see, the strategies that work best for a monopolist differ dramatically from the implied strategies in a competitive situation (in ways that are not intuitively obvious).

For purposes of the monopoly case, assume that demand is linear, as is the externality function. Let $t \in \{a, b\}$ designate the product type. Let $c_m{}^a, c_m{}^b$ be the monopolist's marginal costs for products $a$ and $b$ (and subscripts, such as $_m$ for monopolist, will be suppressed when no possible ambiguity can result).

Let the cross-market externality function be given by $z(q^b) = kq^b$. And let us consider what *k* represents and the degree to which it can be manipulated. To motivate this discussion, let us consider the situation of RealNetworks, which makes both the RealPlayer plug-in (product *b*) and the RealPlayer server (product *a*). It is not unreasonable to assume that the utility of the RealNetworks server to an Internet broadcaster (such as Broadcast.com) is a function of the product's attributes (what the server can do) and how many people the server can broadcast to. (Think of the number of consumers who use the RealPlayer plug-in as $q^b$). Now suppose, RealNetworks modifies the software to provide better information, diagnostics, automated survey information, or whatever, on the population $q^b$. It is not unreasonable to suppose that this better information regarding the population in $q^b$ allows the server to be sold at a higher price, even if $q^b$ (the number of plug-in users) is unchanged. This would be an example of a manipulation of *k.* Anecdotal evidence suggests that web sites can (and do) attempt to increase own *k.* For example, most leading search engines "sell" keywords to advertisers. (Think of advertising as product *a* and the web site content as product *b*). Hence, it is interesting to consider the results of changes in *k* on firm and industry profits.

The demand constants for the $a,b$ markets are $R^a, R^b$, respectively. For the moment assume, without loss of generality, $R^a = R^b = 1$ and $c^a = c^b = 0$.

So, the demand functions are:

$$\begin{aligned} &D^b\left(p^b\right) = 1 - p^b \\ (2)\quad &D^a\left(p^a, p^b\right) = 1 + z\left[D^b\left(p^b\right)\right] - p^a = 1 + kD^b\left(p^b\right) - p^a \end{aligned}$$

Assume for the moment that there are two separate firms acting as monopolists in the two "separate" markets, and these firms are denoted by the subscript $_s$. Unless otherwise noted, annual fixed costs $\left(F^t\right)$ are normalized to zero. The profit functions for these respective firms are given by:

$$\begin{aligned} &\max_{p^b} p^b\left(p^b\right) = \left(p^b - c^b\right)(R^b - p^b) \\ (3)\quad &p_s^{b*} = \frac{1}{2} \\ &p_s^{b*} = \frac{1}{4} \end{aligned}$$

$$\max_{p^a} \pi^a\left(p^a, p^b\right) = \left(p^a - c^a\right) D^a\left(p^a, p^b\right)$$
$$p_s^{a*} = \frac{1}{2}\left(1 + k - kp^{b*}\right)$$
$$\frac{\partial p_s^{a*}}{\partial k} = \frac{1 - p^b}{2}$$
$$\text{(4)}\quad \pi_s^{a*} = \frac{1}{4}\left[-1 + k\left(-1 + p^b\right)\right]^2$$
$$\frac{\partial \pi_s^{a*}}{\partial k} = \frac{1}{2}\left[-1 + k\left(-1 + p^b\right)\right]\left[-1 + p^b\right]$$
$$\pi_s^{a*} = .0625\left(2 + k\right)^2 \left(\text{assume } p^{b*} = \frac{1}{2}, \text{ see above}\right)$$
$$\frac{\partial \pi_s^{a*}}{\partial k} = .25 + .125k$$

And, assuming we have only one firm acting as a monopolist in both markets (designated by subscript $m$), we have:

$$\text{(5)}\quad \begin{aligned} &\pi_m\left(p^a, p^b\right) = \pi^a\left(p^a, p^b\right) + \pi^b\left(p^b\right) \\ &s.t.: p_m{}^b \geq 0 \end{aligned}$$

This yields a corner solution (i.e., $p_m{}^b = 0$) of:

$$\text{(6)}\quad \begin{aligned} &p_m{}^{a*} = \frac{1+k}{2}, p_m{}^{b*} = 0 \\ &\frac{\partial p_m{}^{a*}}{\partial k} = \frac{1}{2} \\ &\pi^*_{\text{m}} = \frac{1}{4}\left[\left(1 + k\right)^2\right] \\ &\frac{\partial \pi^*_{\text{m}}}{\partial k} = \frac{1 + k}{2} \end{aligned}$$

The internal solutions for the monopolist are:

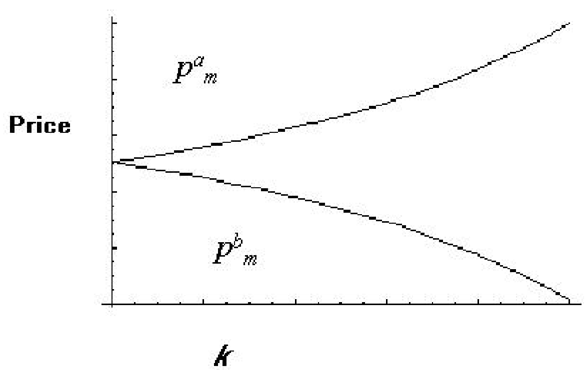
$$p_m^{a*} = \frac{1}{2-k}, \frac{\partial p_m^{a*}}{\partial k} = \frac{1}{(k-2)^2}$$

$$(7) \quad p_m^{b*} = \frac{k-1}{k-2}, \frac{\partial p_m^{b*}}{\partial k} = -\frac{1}{(k-2)^2}$$

$$p_m^{*} = \frac{1}{2-k}, \frac{\partial p_m^{*}}{\partial k} = \frac{1}{(2-k)^2}$$

Note that, as we would expect, $k = 0 \Rightarrow \{p_s^{a*} = p_m^{a*}, p_s^{b*} = p_m^{b*}\}$.

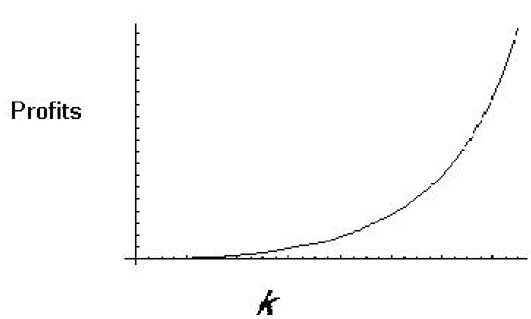
Figure 1 Prices for Various Levels of ***k.***




*1. Proposition (Joint Monopoly): $\frac{\partial p^{b*}}{\partial k} \leq 0, \frac{\partial p^{a*}}{\partial k} > 0$, in joint production, the price of product b is declining in k and the price of product a is increasing in k.*

This is immediate from the comparative statics in *k,* in equations 6 and 7. The intuition is that, as we increase *k,* sales of product *b* become more valuable, and it pays to decrease the price of one product to increase demand for the other. See Figure 1.

Figure 2 Profits of Joint Monopolist Optimum Less Sum of Separate Optimums at Various Levels of *k*.




*2. Proposition (Monopoly): Coordinated pricing of the two products is dominant (i.e., more profitable, $p_m^* \geq (p_s^{a*} + p_s^{b*})$), and the advantage is increasing in **k**. See Figure 2.*

Recall the basic double marginalization problem as discussed by Jeuland and Shugan (1983) and Moorthy (1987). A manufacturer can sell a product through a manufacturer-owned store or an independent retailer. It is well-established that if the firm sells through the independent retailer, profits are not maximized since the retailer selects too high a retail price. The result in the case of a cross-market network externality is strikingly analogous. If the two products $\{a, b\}$ are produced by two separate firms, the firm producing product *b* will sell it at an inefficient (i.e., too high) price.

To demonstrate this for the cross-market externality, consider what the First Order Conditions imply for the monopolist to be at an optimum:

$$(8)\quad \begin{aligned} \frac{\partial p_m}{\partial p_m^a} &= 0 = \frac{\partial p^a}{\partial p_m{}^a} + \frac{\partial p^b}{\partial p_m^a} \\ \frac{\partial p_m}{\partial p_m^b} &= 0 = \frac{\partial p^a}{\partial p_m{}^b} + \frac{\partial p^b}{\partial p_m^b} \end{aligned}$$

Since both $\{p^a, p^b\}$ are functions of $p^b$, the First Order Condition of one will be positive, and the other will be negative at $p_m^*$. In other words, $\frac{\partial p^a}{\partial p_m{}^b} < 0, \frac{\partial p^b}{\partial p_m^b} > 0$ at $\{p_m^{a*}, p_m^{b*}\}$. So at the joint optimum, if the two products are produced by two separate firms, the firm producing product *b* will find it optimal to increase $p^b$. Intuitively, this result is most obvious when we consider solutions where $p_m^{b*} < c^b \left(i.e., c^b \cong 0, k >> 0\right)$, since no independent firm producing just product *b* would set a price below marginal costs.

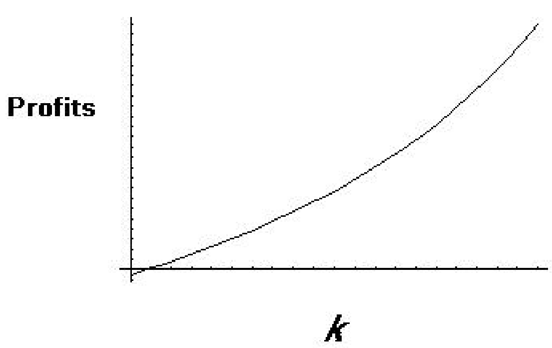
In summary, if the $a, b$ products are produced by separate firms which agree to cooperate, there will be more total profit to divide between them. Consequently, both can gain from cooperation. As with the channel coordination problem, some method of enforcing coordination must exist.

One method of cooperation is to have a single decision-maker dictate all pricing decisions (e.g., by means of joint ownership). But joint ownership may result in several problems. Depending on the situation, the law may frown on joint production of both types of products, or the two types of products may be sufficiently different so as to generate diseconomies when aggregated.

An alternative to joint ownership is a contract specifying each agent's decision variables. For example, as noted in the Introduction, Mitsubishi (rather than purchase a TV network and mandate broadcasts in HDTV format) entered into a contract with CBS, whereby the network agreed to broadcast in the format specified by Mitsubishi in exchange for a payment (subsidy).

Figure 2 illustrates the value of coordination. It is a plot of $\left[p^*_m - \left(p^*_a + p^*_b\right)\right]$ for various levels of $k$. Note that as expected, as $\lim_{k \to 0} p^*_m \to \left(p^*_a + p^*_b\right)$. In other words, with no externality, the individual optimums are also the joint optimum. And we have that $\frac{\partial\left[p^*_m - \left(p^*_a + p^*_b\right)\right]}{\partial k} > 0$. As the cross-market network externality increases, the inefficiency of separate decision-making will also increase.

Figure 3 Monopolist Profits $\left(p^*_m\right)$ as a Function of ***k.***




*3. Proposition (Monopoly):* $\frac{\partial p^*_m}{\partial k} > 0$, *profits are increasing in k.*

This is direct from the comparative statics in Equation 7. Intuitively, as we increase *k*, this increases utility for product *a*, and the monopolist can extract the resulting increase in consumer surplus. See Figure 3.

Table 2 Summary of Subsidiary Results in Monopoly Case

| RESULT | COMMENTS |
|---|---|
| $\left\{ \frac{\partial p_m^{a*}}{\partial c^a} > 0, \frac{\partial p_m^{a*}}{\partial c^b} < 0, \frac{\partial p_m^{b*}}{\partial c^a} > 0, \frac{\partial p_m^{b*}}{\partial c^b} > 0 \right\}$ | Prices are increasing in all marginal costs, except that $\frac{\partial p^{a*}}{\partial c^b} < 0$. Note the importance of the cross-market interaction in explaining these results. In general, increasing marginal costs should lead to increasing prices for a monopolist. This is true in our case, except for $\frac{\partial p_m^{a*}}{\partial c^b} < 0$. Increasing marginal costs for product *b* leads to an increased price (and ceteris paribus, lower demand (i.e., sales)) for product *b*. Lower sales for product *b* means it has lower utility to purchasers of product *a*, which results in a lower equilibrium price for product *a*. |
| For sufficiently large k and small $c^b$, we have $p^b < c^b, p^b < 0$. | If the externality is large enough, at some point it pays to suffer a loss on the sale of each product *b* in order to generate demand in product *a*. This result is directly analogous to (Chaudhri, 1998) where it was found that newspaper publishers (subject to certain conditions) have an incentive to sell newspapers below marginal costs, so as to increase the price at which advertising can be sold. |

The prior results have assumed marginal costs are equal to zero. Many industries that are of interest (such as Television, or firms such as FreePC[7]), do not have low marginal costs. Some subsidiary results regarding marginal costs in joint monopoly are summarized in Table 2.

In summary, we have found that joint production of *a,b* is advantageous for the manufacturer. Yet, this is a puzzle. We find that in certain industries the production of both products is almost always joint. For example, in the print media we find that the newspaper is almost always the producer of the news and the seller of advertising. As we move our analysis to the level of network television, we do not find that the manufacturer of the television sets is naturally the television broadcaster. Other factors seem to be at work. If the model is correct, coordinated

[7] FreePC (www.FreePC.com) "gives" a PC to consumers in exchange for the right broadcast advertising, information collection, etc. Or, in the notation of the model above, the cross-market externality in *k* is sufficient to warrant the giving away of a PC, even though the PC has a substantial marginal cost.

production should be dominant. Casual empiricism indicates, however, that TV stations and TV manufacturers are rarely jointly owned. A final theory of marketing strategies for these types of markets must explain when joint production will be beneficial, and (perhaps more importantly) when it will not. The next section will demonstrate that, in the presence of competition and compatibility, joint production is not necessarily advantageous.

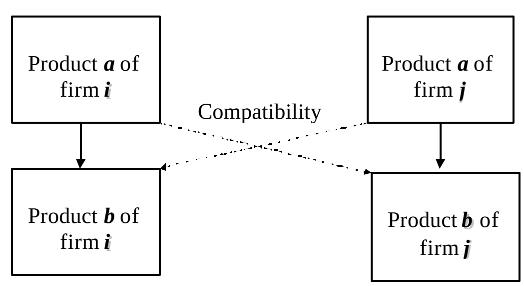
## 5 Cross-Market Externality and Product Compatibility in a Competitive Market

The prior discussion of the monopoly solution provides a framework and some interesting intuition, particularly in certain software markets where monopolies often appear to result from high fixed costs, large sunk costs, and very low marginal costs of production, as well as network effects. However, many other markets of interest cannot be characterized as monopolies. This section uses the Hotelling Line to explore the issues of cross-market externality and compatibility in the context of competition.

The Hotelling Line, first introduced by Hotelling (1929) with Nash equilibrium solutions provided by d'Aspremont, Jaskold Gabszewicz et al. (1979), is a simple model of product differentiation in a location space. Customer location on the lines represents preferences about some attribute, and firm locations represent product differentiation. The advantages to this approach are the clear closed-form solutions that can be achieved. The disadvantages are that the results involve only two firms. As a consequence, a location model is not appropriate for many markets, and the Hotelling line is (by definition) a one-dimensional model. In summary, the Hotelling model (while convenient for theoretical work) is not appropriate for empirical work. The Appendix introduces a model based on a logit demand system (which will be used for the empirical work), and provides a numerical example suggesting that the conclusions presented below are robust to a more complex product-space. The disadvantage of the logit model is the difficulty in providing simple closed-form solutions.

We have previously seen that, in the monopoly case, profits increase in the cross-market externality $(k)$. This result is intuitively appealing since the increased cross-market externality leads to increased consumer utility. Accordingly, we would expect increasing profits in the presence of a monopoly supplier. *However, in a competitive market, we find that increasing the cross-market externality leads to decreasing (or at best, unchanging) profits.*

Figure 4
Compatibility




As we introduce competition to the model, we also need to consider the issue of product compatibility. For our purposes, compatibility $(0 \le C_i \le 1)$ is expressed as the degree to which product $a$ of firm $i$ can communicate with product $b$ of firm $j$ (and vice versa).[8] See Figure 4.

Conventionally, we would expect that increasing compatibility lowers product differentiation, resulting in reduced profits (in the absence of other factors). However, as will be shown below, in markets with a cross-market effect, increasing compatibility (hence, decreasing differentiation) leads to increasing profits.

---

[8] As an example, consider the extent to which a file generated by Microsoft Media Player can be listened to using a RealAudio Plug-in.

This section formally proves the two results described above and provides an economic interpretation. We have two firms, and an arbitrary firm is designated by the subscript $i \; (i \in \{0,1\})$. Each firm produces two products, designated by the superscript $t \; (t \in \{a,b\})$. Type A consumers consume product $a$, and Type B consumers consume product $b$. Reservation utility is $r$ (and is the same for both types of consumers), and demand is inelastic. Consumers are distributed uniformly along two lines of length one, with one type of consumer to each line. Location on the line represents the consumer's preference about some product attribute. The firms must decide where to locate along the line, so for example, $y_0^a$ is the location of firm 0 regarding production of product $a$. After selecting where to produce, the firms must then decide what price to set. So, for example, $p_0^a$ is the price of product $a$ for firm 0. Further, let $D_i^t$ be the demand for product $t$ produced by firm $i$. Without loss of generality, assume $y_0^a \le y_1^a, y_0^b \le y_1^b$. A variable name (without superscript or subscript) represents all the instances of that variable when no ambiguity can result (provided, of course, the variable is not a scalar); so $p$ is a shorthand expression for $\{p_0^a, p_0^b, p_1^a, p_1^b\}$.

Utility for Type B consumers is a function of the price of product $b$, and the square of the distance from the consumer's location (or, if you prefer, the consumer's ideal point) to the firm's location. Therefore, for an arbitrary consumer of product $b$ located at $x$, the utility of firm 0's offering is given by $r - t(y_0^b - x^b)^2 - p_0^b$ (where $t > 0$ represents the consumer's disutility of distance from the consumer's ideal product point). For convenience, assume that both types of consumers have exactly the same value for $t$. For ease of exposition, assume $t = 1$, without loss of generality.

Utility for Type A consumers is a function of the price of product $a$, the square of the firm's distance from the consumer's ideal point, and an externality in the sales of product $b$. The externality is measured by $k$ and (for the moment) assume $k$ is a scalar and is identical for both firms. The degree of compatibility is measured by $C_i$, and we assume $C_0 = C_1 = C$ (again, assumed to be identical for both firms). For example, for an arbitrary consumer of product $a$ located at $x$, the utility of firm 0's offering is given by: $r - t\left(x^a - y_0^a\right)^2 - p_0^a + kD_0^b + CkD_1^b$. Note that the externality in the rival firm's actions is a function of the degree of compatibility.

A Game in a Hotelling space is often conceptualized as a two-stage Game. The first stage is a Game in Location (where each firm selects $y_i^t$), and the second is a Game in Price. For purposes of this section, I assume the Location Game has been played once with the results of: $y_0^a = y_0^b = 0, y_1^a = y_1^b = 1$, and that thereafter locations are fixed. Recall that this is the general solution of a Location Game on the Hotelling line with a quadratic cost function in distance (d'Aspremont, Jaskold Gabszewicz et al. (1979)). The Appendix includes a sketch of a proof demonstrating this as the solution of the Location Game for this particular problem (at the internal solution and at the corner solution).

Therefore, the demand functions are:

$$
\begin{aligned}
&D_0^b[p,y] = \frac{p_0^b - p_1^b + (y_0^b)^2 - \left(y_1^b\right)^2}{2\left(y_0^b - y_1^b\right)} = \frac{1}{2}\left(1 - p_0^b + p_1^b\right) \\
(9)\quad &D_1^b[p,y] = 1 - D_0^b[p,y] \\
&D_0^a[p,y,k,C] = \frac{1}{2}\left(1 - p_0^a + p_1^a - kp_0^b + Ckp_0^b + kp_1^b - Ckp_1^b\right) \\
&D_1^a[p,y,k,C] = 1 - D_0^a[p,y,k,C]
\end{aligned}
$$

The profit functions, assuming that marginal and annual fixed costs are equal to zero, are:

$$
(10)\quad
\begin{aligned}
&p_0^a[p,z,k] = p_0^a D_0^a[p,y,k,C], p_0^b[p,y] = p_0^b D_0^b[p,y] \\
&p_0[p,z,k,C] = p_0^a[p,y,k,C] + p_0^b[p,y] \\
&p_1^a[p,z,k,C] = p_1^a D_1^a[p,y,k,C], p_1^b[p,y] = p_1^b D_0^b[p,y] \\
&p_1[p,z,k,C] = p_1^a[p,z,k,C] + p_1^b[p,y]
\end{aligned}
$$

Each firm seeks to:

$$
(11)\quad
\begin{aligned}
&\max_{p_0^a, p_0^b} p_0[p,y,k,C] \\
&s.t.: p_0^b \geq 0 \\
&\max_{p_1^a, p_1^b} p_1[p,y,k,C] \\
&s.t.: p_1^b \geq 0
\end{aligned}
$$

Recall that I am imposing a non-negativity constraint on $p_i^b$ (to avoid the Moral Hazard and monitoring problems implicit in a negative price). Solving for the appropriate First Order Conditions, we have an internal equilibrium with:

$$
(12)\quad
\begin{aligned}
&p_0^a = p_1^a = 1, p_0^b = p_1^b = 1 + (-1 + C)k \\
&p_0^* = p_1^* = \frac{1}{2}\left(2 + (-1 + C)k\right) \\
&\frac{\partial p_i^*}{\partial k} = \frac{1}{2}(C - 1) \\
&\frac{\partial p_i^*}{\partial C} = \frac{k}{2}
\end{aligned}
$$

And the corner solution is:

$$
(13)\quad
\begin{aligned}
&p_i^a = 1, p_i^b = 0 \\
&\frac{\partial p_i}{\partial k} = 0, \frac{\partial p_i}{\partial C} = 0
\end{aligned}
$$

This yields two propositions, which are of particular interest:

4. *Proposition: If $k_1 = k_0 = k$ (i.e., cross-market externality is the same for all firms, and changes uniformly in the industry), profits are weakly decreasing in k $\left(i.e., \frac{\partial p^*_i}{\partial k} \le 0\right)$ and $\frac{\partial p^*_i}{\partial k}$ is approaching 0 in C $\left(\frac{\partial^2 p^*_i}{\partial k \partial C} = \frac{1}{2}\right)$.*
5. *Proposition: If $C_0 = C_1 = C$ (i.e., compatibility is the same for all firms, and changes uniformly in the industry), profits are weakly increasing in C $\left(i.e., \frac{\partial p^*_i}{C} \ge 0\right)$.*

Note that these propositions arise directly from Equations 12 and 13. First, let us consider how odd these conclusions are. As can be seen in the equations above, $k$ feeds directly into consumer utility. Increasing the cross-market network externality leads to increasing utility for the Type A consumers, but (surprisingly) also leads to lower industry profits. And increasing compatibility normally means lower product differentiation, which typically results in lower industry profits. In this market, however, increasing compatibility results in higher profits.

Intuitively (with regard to the Price Game in the product $a$ product-space), if both firms increase their $k$ simultaneously, the increases cancel each other out. Increasing the **industry** cross-market externality does not change prices or market share in the product $a$ product-space. However, increasing such cross-market externality makes it beneficial for both firms to decrease prices in the product $b$ market (more formally, we have that $\frac{\partial p^b_i}{\partial k}, \frac{\partial p^b_i}{\partial k} \le 0$). Falling prices in the product $b$ market, and competitive pressures (which prevent increased prices in the product $a$ market) cause declines in industry profits in the presence of an increasing industry cross-market network externality.

With regard to compatibility, increased compatibility leads to: (1) a relatively decreasing externality in demand for firm $i's$ own product $b$, and (2) increasing externality in demand for the competitor's product *b*. As a result, the firm has a declining incentive to "subsidize" sales of product *b.* Or, to give an example, to the extent that any firm can broadcast to a RealPlayer plug-in, RealNetworks has less of an incentive to give-away the Plug-in. More formally, we have that $\frac{\partial p_i^b}{\partial C} \geq 0, \frac{\partial p_i^b}{\partial C} \geq 0$.

Figure 5 Change in Firm Profits for Different Strategies in *k*.*

| | Increase *k* | Decrease *k* | Don't Change *k* |
|---|---|---|---|
| **Increase k** | (-,-) | (+,-) | (+,-) |
| **Decrease k** | (-,+) | (+,+) | (-,+) |
| **Don't Change k** | (-,+) | (+,-) | (0,0) |

*Dominant Strategies are highlighted.

We have previously assumed $C_0 = C_1 = C, k_0 = k_1 = k$; now suppose that we allow $C_i, k_i$ to move independently. We find that $\frac{\partial p_i}{\partial k_i} \geq 0, \frac{\partial p_i}{\partial C_i} \geq 0$ (a sketch of this proof is shown in the Appendix). The result in the cross-market externality term is particularly interesting, since it yields a Prisoner's Dilemma in the cross-market externality $(k)$.

In the prior section, I noted that (in the presence of a cross-market externality), we expect to find joint production as dominant. Intuitively, a cross-relationship clearly exists between TV sets and broadcast television, but we don't generally find TV manufacturers jointly-owned with network television. In general, televisions are fully compatible between all TV networks, VCR's, etc. For this reason, the "net" cross-market externality is nearly 0, and there is consequently no advantage to joint-ownership. More implications from these two propositions will be suggested in the Managerial Implications section.

## 6 Casual Empiricism

The prior sections have made several strong predictions. The companion empirical paper will test these propositions using a discrete choice model, and allow for multiple firms with non-identical product characteristics. However, some of the pricing propositions can be tested simply and directly. In particular, in a market where $c_n{}^b \cong c_n{}^a \cong 0, k_n >> 0, C = 0$ and with several competitors, we would expect $p_n{}^b \cong 0, p_n{}^a >> 0$. As more fully explained below, an example of such a market consists of Internet Browser Plug-ins.

Consider Plug-in software and make the conventional assumption that it has low marginal costs. Commercial web sites have a strong economic interest in reaching customers, and should be willing to pay for products that facilitate achieving this goal. Consumers certainly have an interest in viewing web sites, but (given the available universe of web sites) have less of an incentive to pay for a

*Table 3 Representative Pricing for Browser Plug-ins (from Netscape, as of November 1998).*

| Name | Purpose | $p^b$ | $p^a$ |
|---|---|---|---|
| Rubberflex | GIF Animations | 0 | \$100- \$1,000 |
| Auraline Multimedia | MultiMedia | 0 | \$50 |
| Cult3D | 3D Animation non-WRML | 0 | \$100 |
| RealAudio[1] | Streaming Audio | 0 | \$50-\$10,000 |
| Adobe | PDF | 0 | \$300 |

facilitating product. It is not unreasonable to believe that the cross-market network effect will be asymmetric. A stylized analysis of my prior results suggests that we should expect the Plug-in to be given away and jointly produced with the other product.

As of November 1998, the Netscape site listed 176 Plug-ins. All the Plug-in software I located followed the same pricing format. The Plug-in is given away to the consumer, and the other piece of software is sold to web site managers. In addition, the manufacturer of the Plug-in is always the sole manufacturer of the other half of the composite product. Compatibility between rival Plug-ins is low, or close to 0. Some representative prices and firms are shown in Table 3 above. A recent (October 1999) spot check of these prices suggests that little has changed in the relative relationship of these prices.

## 7 Managerial Implications

The managerial implications of these results are both normative and descriptive. Descriptively, these results provide new insights concerning certain categories of computer software. The results on the cross-market term provide a formalized explanation of why firms (such as Adobe, RealNetworks, and others) are willing to spend substantial amounts of money on give-away products (the Plug-ins) that have positive consumer utility.

These results also provide a model indicating under what circumstances firms will offer free computers and ISP service. However, increasing cross-market externality in the presence of competition is almost a marketer's "narcotic"; it provides a short-term increase in profits, but at the long-term price of destroying industry profitability as competitors also increase their own $k$.

The results on compatibility normatively suggest that the "give it away phase" in the Plug-In industry may not last, and that firms such as www.FreePC.com may not have a viable business model. As firms face competitors, they will find that compatibility yields better long-term industry profits. The compatibility results provide a normative ability to examine the evolution of all markets characterized by a cross-market network externality.

The most important message to managers is that they must consider not only what they sell to their own consumers, but also which other products (sold to other consumers) can be used advantageously to generate sales of their product. The examples in this paper have focused on Internet Browser Plug-ins and televisions, but the results are general. Consider some hypothetical examples:

- Education (textbooks and software): Suppose a publisher gives math textbooks and software to a school district. The free material given to the school district is designed so that it provides an enhanced educational experience when used in conjunction with a software product that can be purchased by the parents for home use. Assuming the parents are sufficiently affluent, they will probably purchase the additional enhancing product, and will (in some sense) be locked into this purchase.

- Shopping Agents: These are software packages that "shop" the Internet for consumers. As one example of this type of product, a software firm makes available to shoppers a free software program. The consumer enters data into the program about his/her credit cards, shopping preferences, anniversary dates, and so on. This information remains on the consumer's PC, under the consumer's control. The shopping agent is then used to shop the Internet, based upon the information disclosed by the consumer. In order for the consumer to shop automatically and with anonymity, Internet stores must purchase the other part of this software package. In summary, the shopping agent is given to consumers for free, and the shopping agent server software is sold to firms such as www.barnesandnoble.com, www.amazon.com and other online merchants.

## 8 Future Research

The existing model is abstract and can be extended in various ways. Among these are:

(1) (a) The primary results in this paper are shown using the Hotelling line, but we believe the results can be generalized for all discrete choice and location models with cross-market externalities (provided consumer heterogeneity has certain distribution characteristics); and (b) an explanation can be provided of the empirical observation that product *b* (for example, the Plug-in) is more likely to be the consumer portion of the "bundle".[9]

(2) Characterizing (in a more formal manner) the impact of time (e.g., an infinite horizon setting), sunk costs, switching costs, market growth, and discount rates on these results. In general, I expect that such extended analyses will emphasize the importance of early mover advantages, creating lock-ins, etc. In particular, a strategy evolution is anticipated where competitors begin as monopolists who exploit the cross-market effect, but shift strategies to emphasize compatibility as competitors enter. The issue for the firm is how "competitive" must the market become, before the firm should shift strategy.

(3) Allowing for price discrimination and the product version.

[9] I particularly thank Barry Nalebuff for focusing my attention on these issues and suggesting how the model can be generalized.

(4) Characterizing the issues relating to customer overlap (e.g., how customer purchases from multiple firms would impact the results).

The companion empirical paper explores many of the issues raised in this paper, but primarily focuses on testing the propositions that are derived. As noted above, when the pricing problem is properly formulated, the value of the Lagrangian multiplier will provide useful insights into the value of market strategies that allow the non-negativity price constraint $\left(i.e., p^b \geq 0\right)$ to be violated. This will be explored in an empirical context to suggest the value of couponing, "free" support services, etc. In applications work, we expect the cross-market network externality to run in both directions. The empirical work will allow for this and provide procedures for testing which direction is dominant. Further, the externality function will be allowed to have a more realistic form.

# Appendix

## Various Supplemental Results

### The Location Game

The locations $y_0^a = y_0^b = 0, y_1^a = y_1^b = 1$ are, of course, examples of "The Principle of Maximization" first noted by d'Aspremont, Jaskold Gabszewicz et al. (1979). For our purposes, it is sufficient to demonstrate that the proposed location is a Nash Equilibrium of the Location Game (given the prices we solved for). So, since the proposed solutions are the ends of the lines, we need:

$$(14)\ \frac{\partial p_0^*}{\partial y_0^a}, \frac{\partial p_0^*}{\partial y_0^b} \leq 0, \frac{\partial p_1^*}{\partial y_1^a}, \frac{\partial p_1^*}{\partial y_1^b} \geq 0$$

Substituting the proposed internal solutions $\left\{ \begin{array}{l} \left\{ y_0^a = y_0^b = 0, y_1^a = y_1^b = 1 \right\} \\ Internal: \left\{ p_0^a = p_1^a = 1, p_0^b = p_1^b = 1 + (C-1)k \right\} \end{array} \right\}$ into

the appropriate equations (noting that, for an internal solution, $0 \le C \le 1, 0 \le k < -\frac{1}{C-1}$, for clarity

assume $v = (-1+C)k$, and realizing that $0 \ge v > -1$) results in:

(15)
$$
\begin{aligned}
\left.\frac{\partial p^*_0}{\partial y_0^a}\right|_{\left\{\substack{y_0^a = y_0^b = 0 \\ y_1^a = y_1^b = 1}\right\}} &= \frac{3 + (-1+C)k}{\left[-18 + 4(C-1)^2 k^2\right]} = \frac{3+v}{-18+4v^2} \le 0 \\
\left.\frac{\partial p^*_0}{\partial y_0^b}\right|_{\left\{\substack{y_0^a = y_0^b = 0 \\ y_1^a = y_1^b = 1}\right\}} &= \frac{3 + k - Ck - (C-1)^2 k^2}{-18 + 4(C-1)^2 k^2} = \frac{3 - v - v^2}{-18+4v^2} \le 0 \\
\left.\frac{\partial p^*_1}{\partial y_1^a}\right|_{\left\{\substack{y_0^a = y_0^b = 0 \\ y_1^a = y_1^b = 1}\right\}} &= \frac{-15 + (C-1)k + 4(C-1)^2 k^2}{-18 + 4(C-1)^2 k^2} = \frac{-15 + v + 4v^2}{-18+4v^2} \ge 0 \\
\left.\frac{\partial p^*_1}{\partial y_1^b}\right|_{\left\{\substack{y_0^a = y_0^b = 0 \\ y_1^a = y_1^b = 1}\right\}} &= \frac{-15 + k - Ck + 3(C-1)^2 k^2}{-18 + 4(C-1)^2 k^2} = \frac{-15 - v + 3v^2}{-18+4v^2} \ge 0
\end{aligned}
$$

At the corner solution, for example, we have:

(16)
$$
\begin{aligned}
&\left.\frac{\partial p^*_0}{\partial y_0^a}\right|_{\left\{\substack{y_0^a = y_0^b = 0 \\ y_1^a = y_1^b = 1}\right\}} = -\frac{1}{7}, \left.\frac{\partial p^*_0}{\partial y_0^b}\right|_{\left\{\substack{y_0^a = y_0^b = 0 \\ y_1^a = y_1^b = 1}\right\}} = -\frac{3}{14}, \\
&\left.\frac{\partial p^*_1}{\partial y_1^a}\right|_{\left\{\substack{y_0^a = y_0^b = 0 \\ y_1^a = y_1^b = 1}\right\}} = \frac{6}{7}, \left.\frac{\partial p^*_1}{\partial y_1^b}\right|_{\left\{\substack{y_0^a = y_0^b = 0 \\ y_1^a = y_1^b = 1}\right\}} = \frac{11}{14}
\end{aligned}
$$

**Comparative Statics in** $k_i, C_i$

Assume $\left\{\begin{matrix} y_0^b = y_0^a = 0 \\ y_1^b = y_1^a = 1 \end{matrix}\right\}$. We are assumed to be dealing with myopic firms, and that they are looking at the advantage (or disadvantage) of being able to increase own $k_i, C_i$. Start with the demand functions:

$$(17)\quad \begin{aligned} D_0^b\left[p_0^b, p_1^b\right] &= \frac{1 - p_0^b + p_1^b}{2} \\ D_1^b\left[p_0^b, p_1^b\right] &= 1 + \frac{-1 + p_0^b - p_1^b}{2} \end{aligned}$$

$$(18)\quad \begin{aligned} D_0^a\left[p_i^t, k_i, C_i\right] &= \left\{x : \left\{\begin{matrix} r + k_0\left(D_0^b[\;] + C_0 D_1^b[\;]\right) - p_0^a - (x)^2 = \\ r + k_1\left(D_1^b[\;] + C_1 D_0^b[\;]\right) - p_1^a - (x-1)^2 \end{matrix}\right\}\right\} \\ D_1^a\left[p_i^t, k_i, C_i\right] &= 1 - D_0^a\left[p_i^t, k_i, C_i\right] \end{aligned}$$

The revised profit functions are:

$$(19)\quad \begin{aligned} \pi_0^a\left[p_i^t, k_i, C_i\right] &= p_0^a D_0^a\left[p_i^t, k_i, C_i\right] \\ \pi_0^b\left[p_0^b, p_1^b\right] &= p_0^b D_0^b\left[p_0^b, p_1^b\right] \\ \pi_0\left[p_i^t, k_i, C_i\right] &= \pi_0^a\left[p_i^t, k_i, C_i\right] + \pi_0^b\left[p_0^b, p_1^b\right] \\ \pi_1^a\left[p_i^t, k_i, C_i\right] &= p_1^a D_1^a\left[p_i^t, k_i, C_i\right] \\ \pi_1^b\left[p_0^b, p_1^b\right] &= p_1^b D_1^b\left[p_0^b, p_1^b\right] \\ \pi_1\left[p_i^t, k_i, C_i\right] &= \pi_1^a\left[p_i^t, k_i, C_i\right] + \pi_1^b\left[p_0^b, p_1^b\right] \end{aligned}$$

And the comparative statics in $k_i, C_i$ are:

$$(20)\ \frac{\partial p_0}{\partial C_0} = \frac{k_0 p_0^a \left(1 + p_0^b - p_1^b\right)}{4}$$

$$(21)\ \frac{\partial p_0}{\partial k_0} = \frac{p_0^a \left(1 + C_0 + \left(C_0 - 1\right) p_0^b + p_1^b - C_0 p_1^b\right)}{4}$$

Now assume that at the start of play $\begin{Bmatrix} k_o = k_1 \\ C_0 = C_1 \end{Bmatrix}$, so we have $\begin{Bmatrix} p_0^b = p_1^b \\ p_0^a = p_1^a \end{Bmatrix}$. Hence, $\frac{\partial p_0}{\partial C_0}, \frac{\partial p_0}{\partial k_0} \geq 0$, we have that firm profits for a myopic firm are increasing in $k_0, C_0$, when they start from the same position. The results for the other firm are identical and omitted, and the other cases are analogous and omitted.

**Bertrand-Nash Equilibrium with Differentiated Products**

The main results in the paper are derived using a one-dimensional, two-firms location model that is not suitable for empirical work. The purpose of this section is to: (1) lay the foundation and justification for the discrete choice model that will be used in the companion empirical paper, (2) show that a Nash Equilibrium solution in price will exist with this discrete choice model, and (3) suggest (with a simple numerical example) that the key results derived using the Hotelling line are Nash Equilibrium outcomes using the logit demand system.

The demand system used in this Appendix is a modified multinomial Logit model. A complete derivation for the Multinomial Discrete Choice model can be found in Anderson, de Palma et al. (1992). Formal proofs of the existence of a pure strategy Nash Equilibrium in price for this type of model can be found in Caplin and Nalebuff (1991).

In order to use the multinomial Logit model, it is necessary to define some further notation and to add a few assumptions regarding consumer utility. Assume that both types of consumers are utility maximizers, that each firm's products can be characterized by certain observable characteristics $x_n^t$, and that all consumers of type $t$ value these characteristics with common coefficients $b^t$. The subscript $_{-n}$ means all the elements of the set, except for the $n^{th}$ element. An arbitrary consumer purchasing from firm $n$ at price $p_n^a$ will receive a deterministic utility of:

$$(22)\ u_n^a = b^a x_n^a + z\left[D^b\left(p_i^b\right)\right] - p_n^a$$

Similarly, for product $b$:

$$(23)\ u_n^b = b^b x_n^b - p_n^b$$

The form of $z\left(q_n^b\right) = k_n q_n^b + \sum_{-n} k_n C_n q_{-n}^b$. However, assume that each firm has unobservable attributes, and that the consumer's valuation of these unobserved attributes is uncertain (from the perspective of the firm). In other words, a consumer drawn at random will value the product more (or less) compared to the deterministic utility shown above. Marketing campaigns are assumed to be able to increase or decrease consumer uncertainty (for example) by making product comparisons easier or harder.

The measure of consumer heterogeneity, or (for our purpose) consumer uncertainty, is $m, a$. Hence, a consumer drawn at random has utility given by:[10]

$$
\text{(24)}\quad
\begin{aligned}
\tilde{u}_i^a &= u_n^a + m a_i \\
\tilde{u}_j^b &= u_n^b + m a_j \\
m &\geq 0
\end{aligned}
$$

Assume $a_i$ has mean 0, with corresponding density function of $f(a_1 \ldots a_n)$ and cumulative density function of $F(\ )$. Assuming the utility of the object is large relative to its purchase price and the number of consumers is "large" (i.e., consumers always purchase and in the aggregate, results are predictable), Anderson, de Palma et al. (1992) showed that for firm $n$, demand for products $a, b$ is given by:

$$
\text{(25)}\quad
\begin{aligned}
D^a(p_n^a) &= M^a \int_{-\infty}^{\infty} f(a) \prod_{j \neq n} F(u^a{}_n - u^a{}_j + a)\, da \\
D^b(p_n^b) &= M^b \int_{-\infty}^{\infty} f(a) \prod_{j \neq n} F(u^b{}_n - u^b{}_j + a)\, da
\end{aligned}
$$

$M^t$ is the size of the market. Note that this form is quite general and can accommodate a number of types of distributions, which need not be iid. A particularly convenient, and widely used, assumption regarding the error terms is that $a$ is iid double exponential, with mean 0 and variance $\frac{p^2}{6}$. Hence,

[10] Alternatively, we could formulate this as a random coefficients model. In such a case, marketing efforts might increase or decrease the variance of the distribution of the coefficients.

the variance of the resulting expression is given by $\frac{m^2 p^2}{6}$, where $m$ is a measure of uncertainty. This allows us to represent the demand functions as:

$$
\begin{aligned}
D^a\left(p_n^a\right) &= M^a \frac{Exp\left(\frac{u_n^a}{m}\right)}{\sum_{j=1}^{N} Exp\left(\frac{u_j^a}{m}\right)} \\
D^b\left(p_n^b\right) &= M^b \frac{Exp\left(\frac{u_n^b}{m}\right)}{\sum_{j=1}^{N} Exp\left(\frac{u_j^b}{m}\right)}
\end{aligned}
\tag{26}
$$

In other words, a multinomial logit.[11] Further Caplin and Nalebuff (1991) proved (subject to certain technical conditions) the existence of at least one pure strategy Nash Equilibrium in price for this type of demand system.

Note that as $m \to 0$, the multinomial logit model reduces to the classic deterministic model. As $m \to \infty$, the utilities contain no information and all alternatives become equally likely. I assume a firm can influence customer uncertainty by the nature and type of information distributed (e.g., advertising), and that this will be particularly true for new products. The results of this assumption will be explored in future work.

---

[11] Anderson et al (1992, page 40) remark that if $N \geq 3$, the choice probabilities are given by a multinomial logit if (and only if) the error terms are double exponential. In the case of $N = 2$, actually several distributions produce this result. However, for purposes of this paper and the companion empirical paper, I make the conventional assumption of iid double exponential.

The advantages of modeling demand by a multinomial logit are: (1) the richer and more realistic behavior of the demand functions, and consequently of the agents in the model, (2) the ease of testing theoretical results with actual data (since a multinomial logit model is a flexible and widely used econometric modeling approach), and (3) the “guaranteed” existence of a pure price Nash Equilibrium based on the results in Caplin and Nalebuff (1991). Among the disadvantages of the multinomial logit approach are the difficulties of finding convenient closed-form solutions. Even when closed-form solutions exist, they often lack a simple intuitive interpretation of the resulting algebraic expressions. For these reasons, the core theoretical work has been presented using the Hotelling line.

Assuming N identical firms, the profit functions (where $F_n^t$ are annual fixed costs) are given by:

$$
(27)\quad
\begin{aligned}
&p_n^a = \left(p_n^a - c_n^a\right) D^a \left(p_n^a, p_n^b\right) - F_n^a \\
&p_n^b = \left(p_n^b - c_n^b\right) D^b \left(p_n^b\right) - F_n^b \\
&Max_{p_n^a, p_n^b} p_n = p_n^a + p_n^b \\
&s.t.: p_n^b > 0
\end{aligned}
$$

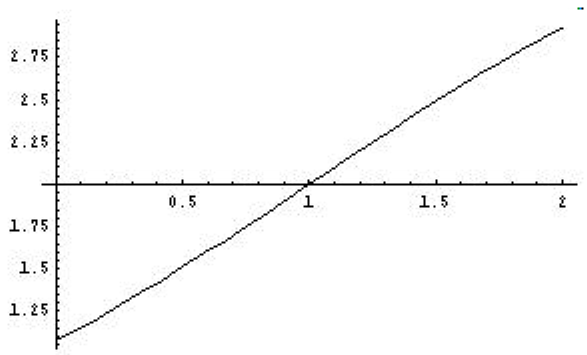
The following observations are relatively direct from the above formulation. As expected $\frac{\partial p_n^{b*}}{\partial k_n} < 0, \frac{\partial p_n^{a*}}{\partial k_n} > 0$, the price of product *b* is declining in *k* and the price of product *a* is increasing in *k*. Hence, coordinated pricing of the two products is favored (i.e., more profitable, $\left(p_n^* \geq \left(p_n^{a*} + p_n^{b*}\right)\right)$), provided $k_n > 0$ and for sufficiently large *k* and small $c_n^b$, we find again that $p_n^b < c_n^b, p_n^b < 0$. Further, as expected $\frac{\partial p_n^{a*}}{\partial c_n^a} > 0, \frac{\partial p_n^{a*}}{\partial c_n^b} < 0, \frac{\partial p_n^{b*}}{\partial c_n^a} > 0, \frac{\partial p_m^{b*}}{\partial c_n^b} > 0$.

Since this is fundamentally a logit model, we find that we have

$$\lim_{m\to\infty} D^a\left(p_n^a\right) = M^a \frac{Exp\left[\frac{u_n^a}{m}\right]}{\sum_{j=1}^{N} Exp\left[\frac{u_j^a}{m}\right]} = M^a \frac{Exp[0]}{\sum_{j=1}^{N} Exp[0]} = \frac{M^a}{N}$$

, market shares become equivalent, and price elasticity declines as uncertainty increases.

Figure 6 Profits Increasing in own *k*




It is central to our argument that $\frac{\partial p_n}{\partial k_n} > 0$, profits are increasing in the firm's own externality, and that if $k_1 = ... = k_n = k$ (i.e., externality is the same for all firms, and changes uniformly in the industry), profits are decreasing in $k$ $\left(i.e., \frac{\partial p_n^*}{\partial k} < 0\right)$.

Figure 6[12] provides a simple numerical example demonstrating that profits are increasing in own *k*.

Hence, we $\exists$ a "Prisoner's Dilemma" in $k_n$. It is individually rational for each firm to try to increase its own *k*, but if all firms increase *k*, the result is decreasing industry profits (See Figure 8). We also

[12] The constants for this section are $s_n^t = 0, m = 1, k_n = 1, c_n^t = 1, g = 0, M^t = 10$, except as otherwise noted. These constants are selected for convenience and simplicity. Other constants will show analogous results. The numerical solutions were found using Mathematica 3.0. Given the assumed values, the Nash Equilibrium was solved by repeatedly applying a Best Response function until the appropriate Fixed-Point of the system was found.

expect to find that we have increasing profits in compatibility, as is shown in Figure 7. Hence, the main results of the paper appear to be robust to a change to a more complex product space. Note that this product space is empirically testable, as it can handle an arbitrary number of firms, product attributes, and so on. Primarily, the empirical work will involve testing for the existence and direction of influence of $C_n, k_n$.

Figure 7 Profits Increasing in Own Compatibility

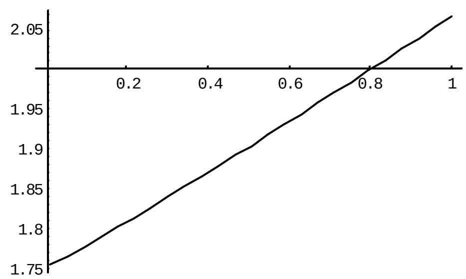
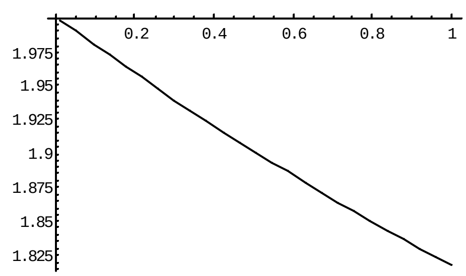



Figure 8 Profits Decreasing in Industry Cross-Market Externality $(k)$




## References

Anderson, S. P., A. de Palma, et al. 1992. Discrete choice theory of product differentiation. Cambridge and London, MIT Press.

Besen, S. M. and J. Farrell 1994. Choosing how to compete: strategies and tactics in standardization. *Journal of Economic Perspectives.* **8** 117-31.

Brinkley, J. May 10 1999. Mitsubishi to underwrite CBS's costs on HDTV. *New York Times.*

Brynjolfsson, E. and C. Kemerer, F. 1996. Network externalities in microcomputer software: An econometric analysis of the spreadsheet market. *Management Science.* **42**(12) 1627-1647.

Caplin, A. and B. Nalebuff 1991. Aggregation and Imperfect Competition: On the Existence of Equilibrium. *Econometrica.* **59**(1) 25-59.

Chaudhri, V. 1998. Pricing and efficiency of a circulation industry: The case of newspapers. *Information Economics and Policy.* **10**(1) 59-76.

d'Aspremont, C., J. Jaskold Gabszewicz, et al. 1979. On Hotelling's "Stability in Competition". *Econometrica.* **47**(5) 1145-50.

Economides, N. 1996. Network Externalities, Complementarities, and Invitations to Enter. *European Journal of Political Economy.* **12**(2) 211-33.

Farrell, J. and G. Saloner 1986. Installed Base and Compatibility: Innovation, Product Preannouncements, And Predation. *American Economic Review.* **76**(5) 940-955.

Gandal, N. 1994. Hedonic price indexes for spreadsheets and an empirical test for network externalities. *Rand Journal of Economics. Spring.* **25**(1) 160-170.

Hanson, W. 1997. Giving it Away: The Optimal Online Enhancement of Traditional Products. *Working Paper*. Stanford University.

Hotelling, H. 1929. Stability in Competition. *The Economic Journal.* **39**(153) 41-57.

Jeuland, A. and S. Shugan 1983. Managing Channel Profits. *Marketing Science.* **2**(3) 239-272.

Katz, M. L. and C. Shapiro 1985. Network Externalities, Competition, and Compatibility. *American Economic Review.* **75**(3) 424-40.

Katz, M. L. and C. Shapiro 1994. Systems competition and network effects. *Journal of Economic Perspectives.* **8** 93-115.

McAfee, R. P., J. McMillan, et al. 1989. Multiproduct Monopoly, Commodity Bundling and Correlation of Values. *Quarterly Journal of Economics.* **104** 371-384.

Moorthy, K. S. 1987. Managing Channel Profits: Comment. *Marketing Science.* **6**(4) 375-379.

Nalebuff, B. 1999. Bundling. *Working Paper*. Yale University.

Schmalensee, R. 1979. Commodity Bundling by Single-Profit Monopolies. *Journal of Law and Economics.* **10** 141-156.

U.S. Bureau of the Census 1997. Statistical Abstract of the United States: 1997 (117th Edition). Washington, DC.

Whinston, M. D. 1990. Tying, Foreclosure and Exclusion. *American Economic Review.* **80**(4) 837-959.